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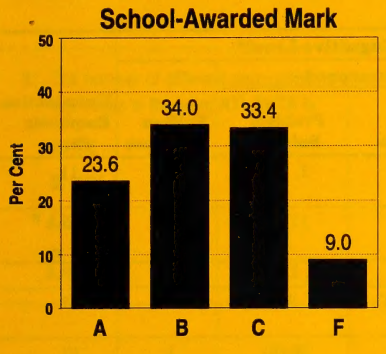
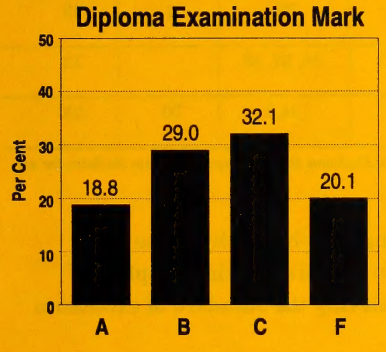
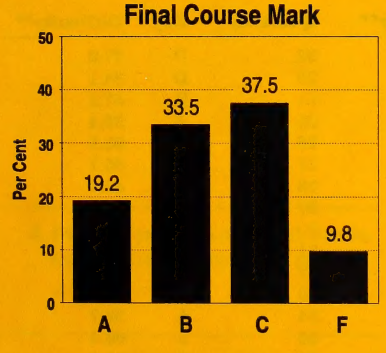
Mathematics 30

Diploma Examination Results
Examiners' Report for January 1993







The summary information in this report provides teachers, school administrators, students, and the general public with an overview of results from the January 1993 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used with the detailed school and jurisdiction reports that have been mailed to schools and school jurisdiction offices. An annual provincial report containing a detailed analysis of the combined January, June, and August results is published each year.

Description of the Examination

The Mathematics 30 Diploma Examination consists of three parts: a multiple-choice section of 42 questions worth 60%, a numerical-response section of 7 questions worth 10%, and a written-response section of 4 questions worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final course marks achieved by 10 609 students who wrote the January 1993 examination.

- 90.2% of these students achieved the acceptable standard (a final course mark of 50% or higher).
- 19.2% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Provincial Averages

- The average school-awarded mark was 67.5%.
- The average diploma examination mark was 63.8%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 66.1%.

During the scoring of the examination, markers expressed concern about the length of time that it took their students to complete the exam. They estimated that students needed an additional 15 to 20 minutes. Based on this, a decision to calculate raw scores out of a total of 60 marks rather than 70 marks was made. Not all students needed more time. In 89 out of 200 schools with 10 or more students writing the exam, at least 85% of students met the acceptable standard before adjustment. However, all results shown on results statements were calculated out of a possible 60 marks.

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Subtest

When analyzing detailed examination results, please bear in mind that subtest results **cannot** be directly compared.

Results are in average raw scores.

Machine scored: 29.6 out of 49

Written response: 8.7 out of 21

Course Content

- Polynomial Functions: 5.1 out of 8
- Trigonometric and Circular Functions: 5.6 out of 13
- Statistics: 7.8 out of 13
- Quadratic Relations: 5.0 out of 9
- Exponential and Logarithmic Functions: 5.3 out of 9
- Permutations and Combinations: 4.0 out of 9
- Sequences and Series: 5.6 out of 9

Cognitive Levels

- Procedural: 9.9 out of 17
- Conceptual: 9.3 out of 15
- Problem Solving: 10.4 out of 17

Note: The written-response section spans all cognitive levels.

Examination Blueprint

Each question on the examination is classified in two ways: according to the curricular content area being tested and according to the cognitive level. The examination blueprint illustrates the distribution of questions in January 1993 according to these classifications. In the machine-scored columns, numbers without parentheses are multiple-choice questions and those in parentheses ( ) are numerical-response questions.

Reporting Category	Questions by Cognitive Level*				Examination Emphasis (%)
	Machine-scored			Written Response	
	Procedural	Conceptual	Problem Solving		
Polynomial Functions	2, 4, 5	3, 6	1, 7, (1)		11
Trigonometric and Circular Functions	10, (2)	8, 9	11, 12	4	18.5
Statistics	13, 18, 19	14, 15	16, 17, (3)	3	18.5
Quadratic Relations	21, 24, 27	20, 22, (4)	23, 25, 26		13
Exponential and Logarithmic Functions	29	28	30 (5)	1	13
Permutations and Combinations	34, (6)	31, 32	33	2	13
Sequences and Series	40, 41, 42	35, 39, (7)	36, 37, 38		13
Examination Emphasis (%)	24.3	21.4	24.3	30	100

*Refer to Appendix D of the 1992–93 *Mathematics 30 Diploma Examinations Program Bulletin* for an explanation of cognitive levels.

This examination has a balance of question types and difficulties reflecting the philosophy of the Mathematics 30 Course of Studies. It is designed so that students capable of achieving the acceptable standard will obtain a mark of 50% or higher and students capable of achieving the standard of excellence will obtain a mark of 80% or higher.

Multiple Choice

For a complete breakdown of student responses by alternative for the multiple-choice questions, please refer to the school and jurisdiction reports.

Question	Key	Difficulty**	Question	Key	Difficulty**
1	C	84.1	22	D	77.0
2	B	68.1	23	D	34.1
3	D	50.9	24	B	57.2
4	C	49.2	25	A	36.4
5	B	58.6	26	A	63.5
6	D	43.6	27	D	65.7
7	A	83.1	28	A	49.4
8	B	78.2	29	C	68.8
9	B	73.0	30	B	64.0
10	D	74.3	31	C	70.9
11	C	68.0	32	A	66.6
12	C	71.8	33	D	51.4
13	A	71.8	34	A	39.7
14	B	70.5	35	B	66.4
15	A	56.7	36	D	70.1
16	C	65.8	37	D	77.8
17	A	35.3	38	D	41.6
18	C	58.1	39	A	83.9
19	B	45.6	40	B	62.8
20	B	61.6	41	C	56.3
21	C	47.1	42	D	70.7

**Difficulty—percentage of students answering the question correctly

Questions 2, 5, 11, 16, 23, 29, 33, and 42 were classified as questions that students who achieve the standard of excellence could successfully answer. Students achieving the acceptable standard were expected to correctly answer the remaining questions. An illustrative question follows:

31. The number of different arrangements of all the letters in the word **ATLANTA** is

A. 12

B. 112

• **C.** 420

D. 5040

Question 31 required students to calculate the number of different arrangements of the letters in the word **ATLANTA**; that is, to calculate the number of permutations with repetitions. Of the students who achieved the acceptable standard but not the standard of excellence, 73% were able to answer this question correctly, as did 92.4% of the students who met the standard of excellence. The knowledge required to answer multiple-choice question 31 could have been used to answer written-response question 2. However, students had a great deal of difficulty applying the concept of permutations with repetitions to the situation in the written-response question.

Numerical Response

Question	Key	Difficulty**
1	5	70.7
2	2.3	71.1
3	95	43.5
4	10.6	53.3
5	18.1	74.6
6	648	27.2
7	22.0	27.2

**Difficulty—percentage of students answering the question correctly

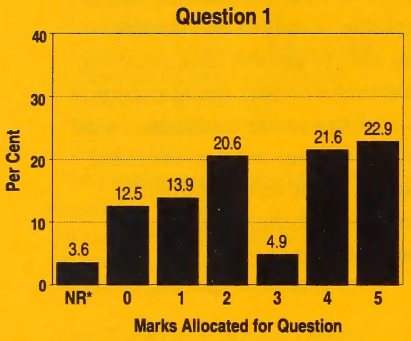
All questions except 5 and 6 were classified as questions that students who meet the acceptable standard should be able to complete successfully. An illustrative question follows:

5. If 6^7 is expressed as a power of 2, then the exponent, correct to the nearest tenth, is _____.

Question 5 required students to use their knowledge of exponential and logarithmic equations to determine the value of an exponent. This question required students to recognize the situation as an exponential equation, derive the equation, and use logarithmic properties to solve for the unknown. Of the students who achieved the acceptable standard but not the standard of excellence, 77.6% met this expectation and arrived at an answer of 18.1. Of those students who met the standard of excellence, 92.9% were able to complete this question correctly.

Written Response

Questions in the written-response section dealt with five of the seven content strands for Mathematics 30. Students performing at the acceptable standard were expected to obtain at least half marks on all questions. Students performing at the standard of excellence were expected to answer all questions almost perfectly. We were very pleased that 98.5% of students attempted the written-response questions.



*NR—No Response

Question 1 required students to determine the number of years that it would take for the population of a town to reach 4800, assuming a constant rate of growth. This question required students to use two concepts: geometric sequence and logarithmic properties. Once students recognized that this growth in population formed a geometric sequence, they then proceeded to determine the *n*th term using logarithmic properties. Students used several strategies for solving this problem. The three most common strategies were writing out all the terms in the sequence, using $t_n = ar^{n-1}$, or using $A = P(1 + i)^n$.

If students carried through their strategy and arrived at an answer of 18.12 . . . years, they received 4 marks. In the looking-back stage of problem solving, students must recognize that they need to make a decision—whether Winterlog would reach a population of 4800 in 18 years or in 19 years. Students need to be able to make such decisions. Just "rounding" the answer to 18 years in fact would not create a population of 4800. Students who completed this looking-back stage achieved full marks. It was expected that students who met the acceptable standard would achieve 4 or 5 marks. Of all students who wrote the examination, 45.5% were able to achieve 4 or 5 marks on this question.

On this 5-mark question, the average mark was 2.71 or 54.2%.

The scoring guide for question 1 was adapted from the Focused Holistic Scoring Point Scale, p. 35, in *How to Evaluate Progress in Problem Solving,* published by the National Council of Teachers of Mathematics, 1987.

Scoring Guide for Written-Response Question 1

5 Marks:

These papers have the following characteristics:

- Appropriate strategies were selected and implemented. The correct answer was given in terms of the data in the problem.

4 Marks:

These papers have one the following characteristics:

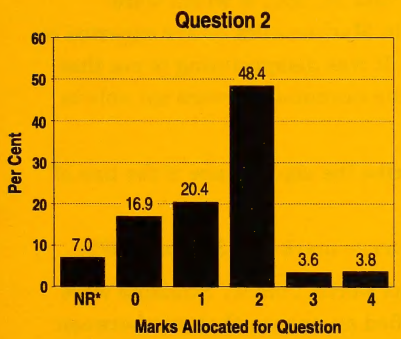
- The student made an error in carrying out an appropriate solution strategy. However, this error does not reflect misunderstanding of either the problem or how to implement the strategy, but rather it seems to be a copying or computational error. The student did look back.
- Appropriate strategies (including the list of calculation of terms in the sequence) were selected and implemented. The answer was not given in terms of the data in the problem.

3 Marks:
These papers have one of the following characteristics:
- The student made an error in carrying out an appropriate solution strategy. However, this error does not reflect misunderstanding of either the problem or how to implement the strategy, but rather it seems to be a copying or computational error. The student did not look back.
- The student has implemented a solution strategy that could have led to the correct solution, but he or she misunderstood part of the problem or ignored a condition in the problem.
- Appropriate solution strategies were properly applied, but—
  a) the student answered the problem incorrectly for no apparent reason;
  b) no answer is given.
- The correct answer is given, and there is some evidence that appropriate solution strategies were selected. However, the implementation of the strategies is not completely correct.

2 Marks:
These papers have one of the following characteristics:
- The student used an inappropriate strategy and got an incorrect answer, but the work showed some understanding of the problem.
- An appropriate strategy was used, but—
  a) it was not carried out far enough to reach a solution
  b) it was implemented incorrectly and thus led to no answer or an incorrect answer.
- The student successfully reached a subgoal, but went no further.
- The correct answer is shown, but—
  a) the work is not understandable;
  b) no work is shown.

1 Mark:
These papers have one of the following characteristics:
- There is a start toward finding the solution beyond just copying data that reflects some understanding, but the approach used would not have led to a correct solution.
- An inappropriate strategy is started but not carried out, and there is no evidence that the student turned to another strategy. It appears that the student tried one approach that did not work and then gave up.
- The student tried to reach a subgoal but never did.

0 Mark:
These papers have one of the following characteristics:
- The data in the problem may be simply recopied but nothing is done with the data, or there is work but no apparent understanding of the problem.
- There is an incorrect answer and no other work is shown.



*NR—No Response

Question 2 required students to show their problem-solving processes in deciding upon the number of bus routes that existed between a department store bus stop and an art gallery bus stop. Students were rewarded for showing their understanding of the problem and then carrying through with a strategy and arriving at an answer. It was expected that students who meet the acceptable standard can contribute toward the problem-solving process for problems involving permutations and/or combinations, including probability problems studied in Mathematics 30. Hence, students who meet the acceptable standard should be able to achieve at least 2 of the 4 marks on this question; 48.4% of the students who wrote the examination met this standard. It was expected that students who meet the standard of excellence would be able to complete the solution to problems that involved the permutations and/or combinations studied in Mathematics 30.
Of all the students who wrote this examination, 3.8% were able to successfully arrive at an answer of 35 routes.

On this 4-mark question, the average mark was 1.43 or 35.8%.

The scoring guide for question 2 was adapted from the Analytic Scoring Scale, p. 30, in *How to Evaluate Progress in Problem Solving*, published by the National Council of Teachers of Mathematics, 1987.

Analytic Scoring Scale for Written-Response Question 2

Understanding the problem

0: Complete misunderstanding of the problem
1: Part of the problem misunderstood or misinterpreted
2: Complete understanding of the problem

Response indicates that there is more than one possible route, where each route consists of going 3 blocks N and 4 blocks E by

- drawing paths on the diagram ***or***
- drawing a tree diagram ***or***
- a written explanation ***or***
- showing mathematical symbols such as ${}_7C_4 \times {}_3C_3$

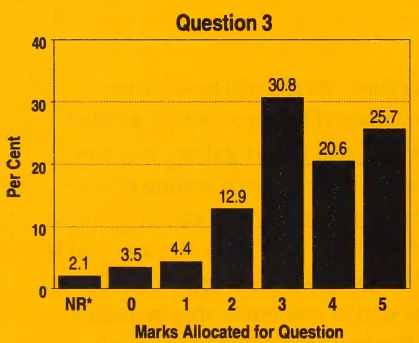
Getting an Answer

0: No answer, or wrong answer based on an inappropriate plan
1: Partial answer for a problem. The student's solution to the problem involves the correct operation for their strategy.

e.g., • If the strategy involves fundamental counting principle, the answer is obtained by multiplying.

• If the strategy involves adding, the answer is obtained by addition.

2: Correct answer



*NR—No Response

Question 3, *parts a* and *b,* required students to read information from the scatter plot and line of best fit, both of which were provided. All students who are in Mathematics 30 are expected to be able to read this information. It was disappointing to see that 10% of the students who wrote the examination were not able to meet this expectation.

Part c required students to describe the significance of the line of best fit in interpreting data.

On this 5-mark question, the average mark was 3.33 or 66.6%.

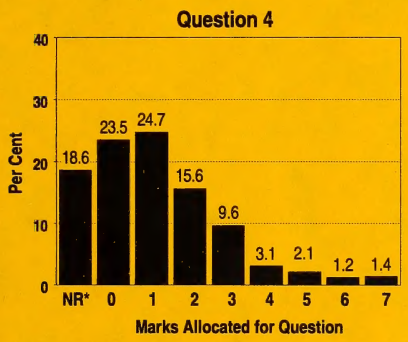
Scoring Question 3: *Part a* was worth 1 mark. Students were awarded the mark if they identified an answer that was between $35.00 and $40.00 inclusive.

Part b was worth 1 mark. Students were awarded the mark if they identified that approximately 5 people could eat breakfast for a total cost of $30. Students were also awarded a mark if they identified 6 people, providing a rationale such as drawing a line on the scatter plot or determining an equation for the line of best fit was included.

Part c was worth 3 marks. The Scoring Guide for *part c* follows:

Scoring Guide for Written-Response Question 3c.

3: The student indicates that the line of best fit in **some way** represents the entire set of data best, not just the individual values.
2: The student provides a feasible scenario for **both** a predicted and an actual cost. For example, the cost could be approximately \$20.00 for 8 people if each person ordered only a coffee and a muffin. The predicted cost for 8 people ranges between \$35.00-\$40.00. Six of the 8 could have ordered the buffet breakfast for \$5.00, the other 2 having coffee and muffins. The student's response somehow relates predicted and actual cost to the line of best fit.
1: The student provides a feasible scenario for **either** the predicted or actual cost.
or
The student's response indicates a correct statement about the line of best fit.
0: The student's work is inappropriate.



Question 4 required students to use their knowledge and understanding of trigonometric and circular functions to algebraically determine the points of intersection for the given graphs. This question tied the whole unit of trigonometry together. The question required students to use knowledge of simplifying trigonometric expressions, of solving trigonometric equations, of analyzing the graphs of trigonometric functions, and in *part b,* of how the parameters affect the intersection of the graphs. Students were given the trigonometric identity needed to start the solution of the equation. Students who meet the acceptable standard were expected to be able to begin the solution of the equation by substituting $2 \sin \theta \cos \theta$ for $\sin 2\theta$ and then to subtract $\cos \theta$ from each side of the equation to arrive at $2 \sin \theta \cos \theta - \cos \theta = 0$. These students achieved 2 out of 5 marks for *part a.* Students at the standard of excellence were expected to carry this through and arrive at the points of intersection. In *part b*, students who perform at the acceptable standard were expected to begin to arrive at an answer by identifying how the amplitude change would affect the graphs.

There have been several questions about why the preamble and the graph were provided for this question. If the question had asked students to find the points of intersection for the graphs of $y = \sin 2\theta$ and $y = \cos \theta$ algebraically, students would have had to know to equate the two equations, know the trigonometric identity for $\sin 2\theta$, and then proceed with the procedure. Those students with a graphing calculator would have had an advantage because they could read the information and even the answer off the graphs. Providing the graphs for all students meant that no student was at a disadvantage. The intent of the question was to see if students could use their skills in algebra to solve this problem and then verify their answers using their knowledge of the graphs of trigonometric functions.

On this 7-mark question, the average mark was 1.25 or 17.9%.

Scoring Question 4: *Part a* was worth 5 marks, which were determined as follows:

Scoring Guide for Written-Response Question 4a.

5: The student has completed the solution using one of the two methods shown. The student's answer may be in decimal form. The θ values may be in degrees.

4: The student has solved the equation by common factoring and obtained **correct** values for θ only, in degrees or radians.

or

The student has solved the equation by dividing by $\cos\theta$ and has determined the points of intersection.

3: The student has solved the equation to obtain $\sin\theta = \frac{1}{2}$ and $\cos\theta = 0$.

or

The student has solved the equation by dividing by $\cos\theta$ and has determined the points of intersection when $\sin\theta = \frac{1}{2}$.

2: The student demonstrates an understanding of the problem and decides on a strategy to solve the problem. For example:

- the student begins to solve the problem by equating the equation $\sin 2\theta - \cos\theta = 0$

or

- the student has solved the equation by dividing by $\cos\theta$ but only found values for θ, where $\sin\theta = \frac{1}{2}$

or

- the student finds a **point** of intersection, other than $\left(\frac{\pi}{2}, 0\right)$

1: The student demonstrates an understanding of the problem by

- substituting $2\sin\theta\cos\theta$ for $\sin 2\theta$ in the equation.

or

- identifying $\theta = \frac{1}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

0: The student's work is inappropriate.

Part b was worth 2 marks, which were determined as follows:

Scoring Guide for Written-Response Question 4b.

2: The student explains how the points of intersection change or do not change.
For example:

- the points of intersection are in the form $(\theta, 3y)$ because the amplitude of both graphs changed from 1 to 3
- the θ values will be the same and the y values will be multiplied by 3
- two of the four points of intersection are common

1: The student demonstrates knowledge that relates to an understanding such as:

- discussing the amplitude change in the graph
- that the θ values remain the same
- that the number of points of intersection will remain the same
- beginning to solve the corresponding equation

0: The student's work is inappropriate.

For further information, contact Florence Glanfield or Phill Campbell at the Student Evaluation Branch, 427-0100.